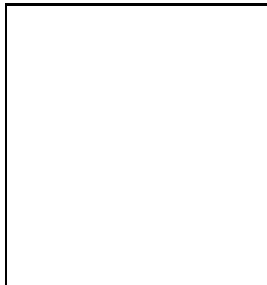


COSMIC STRINGS LENS PHENOMENOLOGY REVISITED

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We present investigations of lens phenomenological properties of cosmic strings for deep galaxy surveys. General results that have obtained for lineic energy distribution are presented first. We stress that generically the local convergence always vanishes in presence of strings although there might be some significant distortions. We then propose a simplified model of strings, we call “Poisson strings”, for which exhaustive investigations can be done either numerically or analytically.

1 Motivations

At a time when the observational data seem to converge towards models of structure formation with scalar adiabatic initial fluctuations obeying Gaussian statistics ¹, it is probably worth recalling that cosmic strings, and more generally topological defects, form under very general circumstances. Therefore, although they may not be the main seeds of the large-scale structure of the universe or of the CMB anisotropies, relics of such objects due to early time phase transitions may still exist. With the advance of a new generation of telescopes and large field CCD cameras we think it is worth to keep in mind what could be their observational effects.

In this presentation we are thus interested in the observational signature of cosmic strings on background galaxies. Future large-scale surveys that are in preparation for weak lensing observations and that are going to cover a fair fraction of the sky with unprecedented image quality are a natural playground for elaborating detection strategies of such strings.

2 General lens properties of strings

2.1 Uniform straight strings

The case of uniform straight string (with Goto-Nambu equation of state) has been exhaustively described in the literature². In this case the metric is actually flat around the string. The only effect of the string is to induce a “missing angle” so that space is conical around the string. As a result galaxies that are behind the string may exhibit double images. The pair separation of these images is directly proportional to the lineic energy density of string μ_0 . For strings that formed at the Grand Unification scale, that would corresponds to image separation of a few arcsecs. Optical investigations, for that respect, seem the most appropriate way of detecting such strings.

However, the idea strings induce series of image pairs is however coming from a simplified description of the string energy properties. Simple numerical experiments³ done with numerical simulations of string networks suggest that this is too naive a view and that the lens phenomenology of strings is much more complex.

2.2 General string effects

In general the displacement field induced by strings is not uniform, and thus significant deformation effects of background galaxies can be induced. One can show that, under very general hypothesis (with respect to the string equation of state and geometry), there exists a potential from which the elements of the deformation matrix derive⁵. It can formally be written in terms of the angular positions (x, y) ,

$$\varphi(x, y) = 4G \frac{D_{\text{LS}}}{D_{\text{OS}}} \int ds \mu[x_{\text{str.}}(s), y_{\text{str.}}(s)] \log \left([x - x_{\text{str.}}(s)]^2 + [y - y_{\text{str.}}(s)]^2 \right)^{1/2} \quad (1)$$

where $(x_{\text{str.}}(s), y_{\text{str.}}(s))$ are the angular string coordinates for the angular curvilinear position s , $\mu(x_{\text{str.}}(s), y_{\text{str.}}(s))$ is the “projected energy density” at those positions, G is the Newton constant and $D_{\text{LS}}/D_{\text{OS}}$ is the ratio of the angular distance between the string and the source-plane to the one between the observer and the source plane in the thin lens approximation. The projected energy density is located on the intersection between the string world-sheet and the past light cone of the observer (e.g. Fig. 2.2). Its amplitude is given by a combination of the projected T_{00} , T_{0z} and T_{zz} components of the stress-energy tensor of the string if the line-of-sight is along the z direction. The displacement field is given by,

$$\xi_i = -\partial_i \varphi(x, y), \quad (2)$$

and the elements of the deformation matrix can be written as,

$$\gamma_1 = \left(\partial_x^2 - \partial_y^2 \right) \varphi(x, y), \quad (3)$$

$$\gamma_2 = 2 \partial_x \partial_y \varphi(x, y), \quad (4)$$

the local convergence being zero except on the string itself. This result holds despite the fact that not only scalar fluctuations are contributing to the lens effects, but also vector and tensor modes⁵.

3 A simplified model: the Poisson strings

In order to have analytical insights into the string lens phenomenology, realistic models for string shape and energy should incorporate complex features: the string are very far from being straight

Figure 1: Schematic view of the source term for the lens effect of a string. It corresponds to the intersection of the loop world-sheet and the past light cone of the observer.

lines with uniform energy distribution. We choose⁴ to describe the energy fluctuation in a simple manner, assuming that the string follows a straight line, but with local energy fluctuations. This fluctuations are assumed to account for the various changes of shape, density of the strings, to possible non-standard equation of states, or to the existence of currents along the string. We therefore assume the string to be straight along the y direction, and the local projected lineic energy density $\mu(s)$ to be a random field. To specify our model we still need to explicit the statistical properties of the μ field. Its average value obviously does not vanish and is given by

$$\langle \mu(s) \rangle = \mu_0 \equiv \frac{\xi_0}{4\pi G D_{LS}/D_{OS}}, \quad (5)$$

where ξ_0 is the typical angular displacement induced by the string ($2\xi_0$ would be the pair separation if the string were uniform).

With the lack of any well understanding of the string microscopic physics, we simply assume the coherence length of μ to be much smaller than the other distances intervening in the problem. In this case the 2-point correlation function of μ can be written,

$$\langle \mu(s_1)\mu(s_2) \rangle = s_0 \mu_0^2 \delta_{\text{Dirac}}(s_1 - s_2). \quad (6)$$

An important consequence of this assumption is that, at finite distance from the string, the deformation matrix elements are sourced by an infinite amount of independent portions of the string. They thus obey Gaussian statistics. The lens properties of the string are thus entirely determined by (6), independently of what could be the one-point probability distribution function of μ .

Elementary phenomenology of "Poisson string"

On Fig. 2 we depict an example of numerical implementation of such a cosmic string, showing various features associated with this lens system. The grey levels show the variation of amplification given by the determinant of amplification matrix. Along the brightest areas the amplification is infinite; these locations form the critical lines. This is where the most dramatic lens effects could be detected: giant arcs, merging of images... Note that such lines simply do not exist for strings with uniform density! The critical lines are made mainly of two long lines running along the string without crossing it.

It is worth noting that we are here generically in a regime of 3 images in the vicinity of the string (except in rare cases where it can be 5) instead of 2 as for a strictly uniform string. The dashed lines show the locations of the counter images of the critical lines. It delimits the region, in the image plane, within which multiple images can be found. It can be noted however that the amplification rapidly decreases in the vicinity of the string, so that central images (i.e. the ones situated in between the two infinite critical lines) are expected to be strongly de-amplified, except when they are close to one of the critical lines.

3.1 Statistical properties

The necessary conciseness of these notes does not allow to present a detailed presentation of the statistical properties of such a system. The simplicity of the model allows however the

Figure 2: Numerical experiment showing the amplification map, i.e. $\det(\mathcal{A})$, of a “Poisson string”. The brightest pixels correspond to infinite magnification: they form the critical lines. The darkest pixels correspond to a magnification close to zero. The solid lines correspond to the caustics, positions of the critical lines in the source plane. The external dashed lines are the counter images of the critical lines.

Figure 3: Shape of the function $\bar{n}_{\text{crit.}}(x)$.

computation of a lot of general properties, from the position of the critical lines, to the total length of the caustics⁴.

In Fig. 3 we present for instance the number density of intersection points of the critical lines with any horizontal axis. From this one can infer for instance the average number of intersection points (on one side of string):

$$\bar{n} = \int_0^{+\infty} dx n_{\text{crit.}}(x) = \frac{2}{\sqrt{3}} \approx 1.155. \quad (7)$$

The number of intersection points being an odd number, it means that the critical line crosses one horizontal line more than once in at most 7% of the cases. It supports the fact that we are dominated by the two long critical lines located on each side of the string. In rare cases, however, inner critical lines can give rise to complex multiple image systems.

The typical distance of the critical lines to the string can also be computed. It is given by

$$d_{\text{crit.}} = \frac{\int_0^{+\infty} dx x \bar{n}_{\text{crit.}}(x)}{\int_0^{+\infty} dx \bar{n}_{\text{crit.}}(x)} \approx 0.70 (s_0 \xi_0^2)^{1/3} \quad (8)$$

whereas the scatter of this distance is about

$$\Delta d_{\text{crit.}} = 0.31 (s_0 \xi_0^2)^{1/3}. \quad (9)$$

The total length of the critical lines (per unit string length) turns out to be also calculable. Remarkably one finds

$$L_{\text{crit.}} = \frac{4}{\sqrt{3}} \mathbf{E} \left(\frac{3}{4} \right) L \approx 2.80 L, \quad (10)$$

where \mathbf{E} is the complete elliptic integral, *independently* of the parameter of the model. In particular it does not depend on the dimensionless ratio s_0/ξ_0 , nor on the position of the source plane! And because this result is finite, it also proves that the closed critical lines have a finite total length despite the fact that they are in infinite number.

4 Observational prospects

In Fig. ?? we present a simulated image of background galaxies deformed by a straight Poisson string (the case of a Poisson string loop is depicted on the cover). On these images pairs can be clearly identified, although distances and orientations fluctuate from pair to pair contrary to the standard picture. One can also notice numerous small images that appear along the string. The presence of these images are due to the fluctuating small scale structures of the string. They are associated with an infinite number of critical lines (and caustics) near the string. In high resolution images, that might be the most effective way of detecting strings.

This investigation provides a new description of the phenomenological properties of lens physics for cosmic strings. The model we present is based on general results which state that

the lens effects of string are those obtained from lineic energy density. The model we propose should capture most of the generic properties expected in such a case. Although we have obviously not demonstrated the validity of the description we adopted, its resemblance with previous numerical results obtained with simulated string networks³ makes us think that this model could serve as a guideline for detection strategies in future large angular surveys.

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